UPISC UNIVERSITY OF PITTSBURGH INFRASTRUCTURE SENSING COLLABORATION WORKSHOP

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Multi-Fidelity Framework for Thermal Conductivity of Al−Cu

INTRODUCTION

- Goal is to optimize sensor locations to maximize prediction accuracy, minimized uncertainty, and used limited sensor.
- LF models are inexpensive but less accurate. HF models are expensive but more accurate.
- LF and HF data modeled as separate Gaussian processes (GPs) with own kernels
- Use square exponential kernel
- Integrating LF and HF improves overall prediction performance.
- LF and HF combined into joint probabilistic model.
- Multi-fidelity modeling aims to leverage low-fidelity (LF) and high-fidelity (HF) data sources.

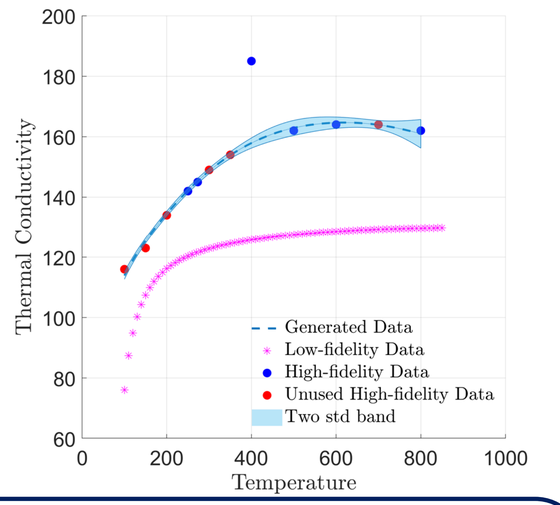
METHODOLOGY

$$u_L(x) \sim \mathcal{GP}(0, k_{LF}(x, x', \theta_L))$$
$$u_H(x) \sim \mathcal{GP}(0, k_{HF}(x, x', \theta_H)$$

$$y_L(x) = u_L(x) + \epsilon_L$$
$$y_H(x) = \rho u_L(x) + \delta(x) + \epsilon_H$$

$$\delta(x) \sim \mathcal{GP}\big(0, k\big(x, x'; \theta_\delta\big)\big)$$

$$\epsilon_L \sim \mathcal{N}\big(0, \sigma_{n_L}^2\big) \text{ and } \epsilon_H \sim \mathcal{N}\big(0, \sigma_{n_H}^2\big)$$



$$\mathbf{K} = \begin{bmatrix} k_{LL}(\mathbf{x}_L, \mathbf{x}_L) + \sigma_{n_L}^2 \mathbf{I} & k_{LH}(\mathbf{x}_L, \mathbf{x}_H) \\ k_{HL}(\mathbf{x}_H, \mathbf{x}_L) & k_{HH}(\mathbf{x}_H, \mathbf{x}_H) + \sigma_{n_H}^2 \mathbf{I} \end{bmatrix}$$

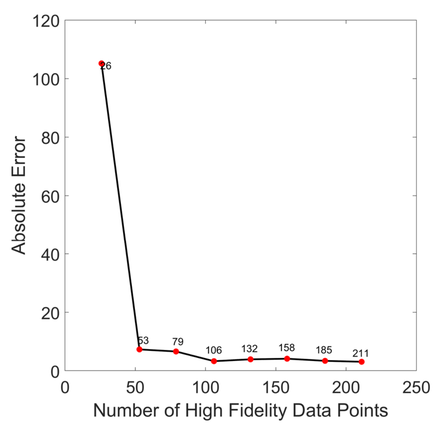
$$k(x, x') = \sigma^2 \exp\left(- \sum_{i=1}^{D} (x_i - x_i')^2 / \theta_i^2 \right)$$

$$k_{LL}(\mathbf{x}_L, \mathbf{x}_L) = k(\mathbf{x}_L, \mathbf{x}_L; \theta_L)$$
$$k_{HL}(\mathbf{x}_H, \mathbf{x}_L) = \rho k(\mathbf{x}_H, \mathbf{x}_L; \theta_L)$$
$$k_{HH}(\mathbf{x}_H, \mathbf{x}_H) = \rho^2 k(\mathbf{x}_H, \mathbf{x}_H; \theta_L, \theta_\delta, \rho) + k(\mathbf{x}_H, \mathbf{x}_H; \theta_\delta)$$

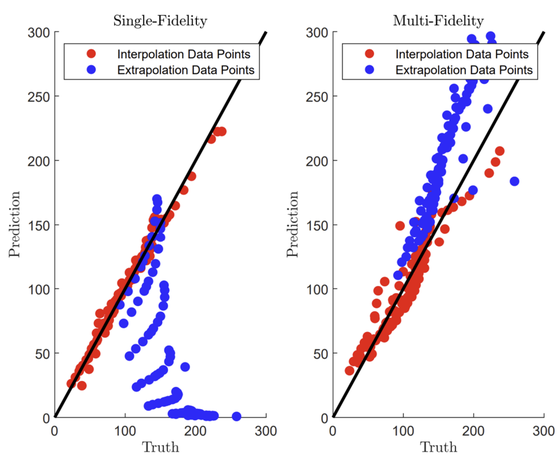
DEMONSTRATION CASE (Al-Cu)

- **Modeling thermal conductivity of Al-Cu, as a function of Al composition and temperature**
- Training multi-fidelity model
  - 80% LF data
  - 20% HF data (set Al composition less than 0.7)
- Testing: 10000 data points (Al composition between 0 and 1)

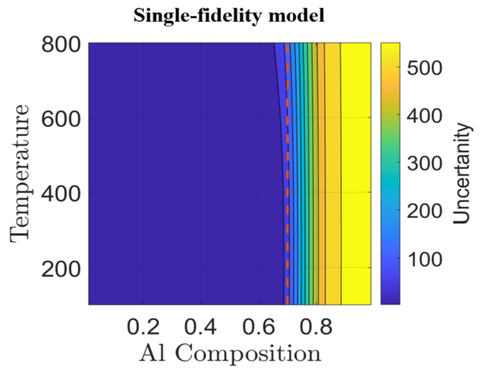
- Training single-fidelity model
  - 20% HF data (set Al composition less than 0.7)
  - Exact same data points as used in the **multi-fidelity** model
- Testing: 10000 data points (Al composition between 0 and 1)



Effect of Number of High-Fidelity Data Points on Accuracy



Models Prediction for Extrapolation Data Points



Uncertainty of thermal conductivity prediction as a function of Al composition and temperature